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A theoretical model of water and trade

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ABSTRACT

Water is an essential input for agricultural production. Agriculture, in turn, is globalized through the trade of agricultural commodities. In this paper, we develop a theoretical model that emphasizes four tradeoffs involving water-use decision-making that are important yet not always considered in a consistent framework. One tradeoff focuses on competition for water among different economic sectors. A second tradeoff examines the possibility that certain types of agricultural investments can offset water use. A third tradeoff explores the possibility that the rest of the world can be a source of supply or demand for a country's water-using commodities. The fourth tradeoff concerns how variability in water supplies influences farmer decision-making. We show conditions under which trade liberalization affect water use. Two policy scenarios to reduce water use are evaluated. First, we derive a target tax that reduces water use without offsetting the gains from trade liberalization, although important tradeoffs exist between economic performance and resource use. Second, we show how subsidization of water-saving technologies can allow producers to use less water without reducing agricultural production, making such subsidization an indirect means of influencing water use decision-making. Finally, we outline conditions under which riskiness of water availability affects water use. These theoretical model results generate hypotheses that can be tested empirically in future work.

1. Introduction

We live in an increasingly globalized world [12,21], where trade in water-intensive commodities, such as agricultural products, represents an important interaction between people and water resources [1]. The relationship between international trade and water resources is an issue of great interest in the literature [11,17]. A number of empirical studies have made reference to classic international trade models, but it is not always clear what the theoretical foundations of the models are and whether they are useful for the case of water [28]. Some studies have argued that economic models are inadequate for explaining virtual water trade [2], while others have sought to clarify the role of economics as it relates to this issue [20]. Many studies focus on the relationship between trade and virtual water resources [10], without direct consideration of domestic, physical water resources. A theoretical model that incorporates domestic water use in production – in addition to the consumption and trade of water-intensive commodities – would contribute to this growing literature. As such, the main goal of this paper is the development of a trade model that addresses these relationships through the explicit inclusion of water resources.

In this paper, we develop a theoretical model designed to emphasize several tradeoffs in water use. First, the model captures competition for water among different sectors. Second, the model allows for the possibility of factor substitutes for water, in the form of alternative production technologies. An example is capital-intensive efficient irrigation technologies and crop varietal improvements, a situation where increased use of one resource (in this case capital) may be able to offset or substitute to some extent for water use. Third, we allow for production and consumption to be substituted across locations in space through trade. Fourth, we explicitly capture farmer risk aversion to variable water supplies, as compared with traditional profit maximizing behavior.

The main goal of our model is to gain generalizable insights into the interactions between people and water in a trading economy. Transferable understanding is often difficult to obtain when more realistic, but heavily parameterized, models are used to inform management of site-specific water resources. Hydro-economics has long been interested in the interactions between people, water resources, and economics [9], though with a focus

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on finding feasible and optimal solutions to concrete problems, i.e. a 'normative' approach to model development [25]. This differs from the development of models in the realm of coupled human and natural systems [18], from which socio-hydrology stems [26], which tend to focus on understanding what is happening in the system and why, following a 'positive' approach to model development [25]. In this way, our model complements existing hydro-economics models, which are typically parameterized to capture local dynamics and inform management [7]. Our modeling approach parallels that of socio-hydrology, yet we help to broaden socio-hydrology through the incorporation of economics. By modeling domestic water use, agricultural production, and trade, we also present a theoretical foundation for the virtual water trade literature. In this way, we aim to contribute to further integration of hydro-economics, socio-hydrology, and virtual water trade research.

The model is inspired by contemporary contexts – such as the current drought in California – where water is a scarce resource. In this setting, in which there is much agricultural production, competition exists between the agricultural sector and other parts of the economy for scarce water resources. Additionally, uncertainty about the future supplies of water resources impacts farmer decision-making. In contexts such as this, it is critical to understand the ramifications of trade in water-intensive goods, as well as how various policies may impact water use, agricultural production, and economic welfare. For this reason, a model that can provide insight into these issues may be of interest to governments, planning authorities, and non-governmental organizations dealing with scarce water resources. However, it is important to recognize that theoretical models are necessarily abstractions of the real world and are not intended to inform policy makers in a specific situation, unlike site-specific integrated water resources management approaches [14].

While the model is inspired by the real world, there is no validation because this is a theoretical model that abstracts the real world with necessarily restrictive assumptions. Our theoretical model is meant to provide a logically consistent framework for deriving results from first economic principles, that is, from the interactions of consumer and producer decision-making. For this reason, we employ many common assumptions of economic modeling, such as equilibrium prices, rational behavior, and profit maximization. These assumptions are pervasive in economic modeling, but rarely exist in the real world, making empirical validation difficult. For this reason, it is common for theoretical economic models to be developed without validation against existing data [6]. However, our model enables us to isolate some of the key parameters that can be empirically estimated in future work. Additionally, our theoretical model generates hypotheses that can be tested with data in specific circumstances in future research.

The approach undertaken in this paper does not involve prediction of bilateral trade patterns among multiple countries; rather it applies to a small, open economy, in which water is scarce. By 'small' we imply that we are concerned with a region that is not so important to international trade that it can significantly influence the prices it pays for inputs and the prices received for outputs; it takes these prices as given. By 'open' we imply that the economy is influenced by supply and demand as reflected in prices received for outputs in the rest of the world. Our approach builds from the traditional two-factor and two-good economic approach associated with Heckscher [8] and Ohlin [19], further refined and extended in Samuelson [22] and Jones [15]. In contrast to these general economics approaches, we allow for water as an additional factor of production, do not assume that capital is perfectly mobile between sectors, and allow for variability in water supplies and hence prices. We go beyond classic studies such as Howitt and Taylor [13] by considering an economy that is open to international trade and has more than one sector, both of which use more than one factor of production. We also relax the traditional profit maximizing assumption to allow for variation in producer attitudes towards risk.

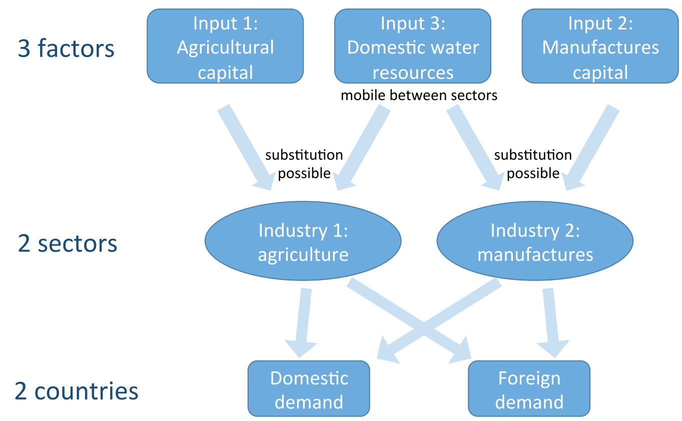

Fig. 1. Schematic of model framework.

The paper is organized as follows. We first develop the model in Section 2. We present two formulations: one that assumes profit maximization and one that enables farmer decision-making under uncertainty. Next, we examine scenarios and policy interventions of interest in Section 3. In Section 3, we ask the following questions: What happens to water use when there is agricultural trade liberalization? What are the consequence of policies to tax water and subsidize water-saving technologies? How does water supply variability impact water use? We conclude in Section 4.

2. Model framework

We develop a theoretical model that captures the water resources tradeoffs outlined above. This model stems from the classic 2 × 2 × 2 trade model, in which there are two regions, two factors, and two goods. The model has explicit treatment of only one small country, but has three factors, one of which is shared by the two sectors. We employ a static equilibrium framework. Equilibrium is reached when prices equilibrate quantity supplied and demanded across all markets in the economy [3]. Representative human agents operate in the model framework according to their objective, which is traditional profit maximization in Section 2.1 and maximization of expected utility under risk in Section 2.2. Under traditional profit maximizing behavior farmers choose among alternative techniques of production based upon the relative prices of inputs. Farmers choose the level of input wherein the price that must be paid for it equals the marginal value product of that input, which is the product of the extra output made possible by one more unit of input (marginal physical product), and the price of the output. This model does not explicitly model multiple regions and make bilateral trade predictions. It is a model of a domestic open economy, in which production and trade are driven by external prices received for goods.

A schematic displaying our model is provided in Fig. 1. We assume that there is a home country and the rest of the world. The country produces two goods: good 1 (agriculture) and good 2 (manufacturing), which are also the two sectors/industries in the economy. There are three factors in the model: factor 1 (agricultural capital), factor 2 (manufacturing capital), and factor 3 (water). Agricultural production requires agricultural capital and water, while manufacturing needs manufacturing capital and water. Water is mobile and costlessly re-allocated between the two sectors, while capital is a specific factor to each sector. Water use links sectors with one another, which is a unique feature of water [23]. In our model, the common demand for water is the principal linkage between the two sectors, since capital is sector-specific. In this way, the model explains intersectoral competition for water resources.

In our model, we allow producers to substitute between factors. This is different from the Leontief assumption – in which factors are assumed to be used in fixed proportions – which is commonly applied when modeling agricultural water use (e.g. Kahil et al. [16] and Berrittella et al. [4]). If farmers adopt efficient irrigation technology, such as drip irrigation, or switch to water-saving crop varietals, this can be thought of as farmers substituting more capital in order to use less water. These substitutions require a monetary outlay, represented in our model as agricultural capital. Ultimately, financial investment enables the same quantity of good to be produced with less water.

The model works for situations in which there is a market for water – in which it is a scarce good – such that obtaining an additional unit of water in one sector requires the other sector to reduce its water use. As such, the model is not intended to depict rainfed agriculture. To keep the analytical framework simple, we assume perfect competition in input (factor) and output (goods) markets. It is assumed that goods are produced with constant returns to scale technology, which means that a doubling of inputs results in a doubling of outputs. Furthermore, the production function is increasing, concave and linearly homogenous, which implies a diminishing marginal product of water and capital. Marginal product refers to the change in output resulting from using one more unit of a particular input, assuming other input quantities are fixed. We assume that inputs receive their value of marginal product, although we will show how this changes under stochastic water supplies.

2.1. Profit maximization

Farmers (or manufacturers) adjust their demand for capital and water so as to maximize profit, which is determined by revenue minus the cost of capital and water per unit crop (manufactured good) produced. Note that capital is specific to each sector, while water is shared between sectors:

$$\max_{x_{11},x_{31}} \Pi_1 = p_1 f_1(x_{11}, x_{31}) - w_1 x_{11} - w_3 x_{31} \tag{1}$$

$$\max_{x_{22},x_{32}} \Pi_2 = p_2 f_2(x_{22}, x_{32}) - w_2 x_{22} - w_3 x_{32} \tag{2}$$

where Π_j is the profit in industry j; p_j is the output price in industry j; $f_j()$ is the production function in industry j; x_{ij} is the unconditional factor demand for factor i in industry j; and w_i is the factor price for input i ($i = 1, 2, 3$ and $j = 1, 2$).

The production function may be any functional form that maintains the following assumptions: (i) it is increasing with respect to either input, (ii) it is concave, meaning that the second derivative is negative, and (iii) it is homogeneous of degree one in inputs, meaning that a doubling of the inputs leads to a doubling of outputs, that is, there are constant returns to scale in the production of each good.

We denote y_j as the output supply in industry j:

$$y_1 = f_1(x_{11}, x_{31}) \tag{3}$$

$$y_2 = f_2(x_{22}, x_{32}) \tag{4}$$

Under constant returns to scale, Eqs. (1) and (2) can be developed in terms of one unit of production. So, producers in each industry choose factors to maximize profit subject to the production technology required to produce one unit of each good:

$$\max_{a_{11},a_{31}} \pi_1 = p_1 - w_1 a_{11} - w_3 a_{31} \text{ subject to } f_1(a_{11}, a_{31}) = 1 \tag{5}$$

$$\max_{a_{22},a_{32}} \pi_2 = p_2 - w_2 a_{22} - w_3 a_{32} \text{ subject to } f_2(a_{22}, a_{32}) = 1 \tag{6}$$

where $a_{ij} \equiv x_{ij}/y_j$ is the conditional factor demand for factor i to produce one unit of good in industry j; π_j is the unit profit in industry j.

The first order conditions can be solved to derive the producer's demand for inputs, conditioned on a desire to produce one unit of output: $a_{11} = a_{11}(w_1, w_3)$, $a_{31} = a_{31}(w_1, w_3)$, $a_{22} = a_{22}(w_2, w_3)$, $a_{32} = a_{32}(w_2, w_3)$. It is important to emphasize that this is a much more realistic approach than to assume that such coefficients are fixed. Assuming that a_{11} and a_{12} are fixed would make the analysis easier to solve later on, but would also simplify things in a somewhat arbitrary manner.

Due to free entry and other competitive market assumptions, marginal revenue (price) equals marginal cost in equilibrium:

$$p_1 = w_1 a_{11}(w_1, w_3) + w_3 a_{31}(w_1, w_3) \tag{7}$$

$$p_2 = w_2 a_{22}(w_2, w_3) + w_3 a_{32}(w_2, w_3) \tag{8}$$

Note that this differs from the Heckshser–Ohlin model (mentioned above) in that w_1 and w_2 are not identical prices for the same input (capital).

We do not need to make consumer preferences or demand curves explicit because of two assumptions. One is that there are competitive market conditions, meaning that the price for any good is equal to its marginal cost of production (anyone who tries to sell for higher than cost will not be able to sell, because someone else will enter and sell at cost). Second, the economy of interest is 'small' relative to the rest of the world, meaning that it is not large enough to affect the prices received for goods produced.

Now, totally differentiate the competitive profit condition and rearrange the equations to obtain the following results (details are in Appendix A):

$$\frac{dp_1}{p_1} = \frac{a_{11}w_1}{p_1}\frac{dw_1}{w_1} + \frac{a_{31}w_3}{p_1}\frac{dw_3}{w_3} \tag{9}$$

$$\frac{dp_2}{p_2} = \frac{a_{22}w_2}{p_2}\frac{dw_2}{w_2} + \frac{a_{32}w_3}{p_2}\frac{dw_3}{w_3} \tag{10}$$

Denoting $\hat{x} \equiv \frac{dx}{x}$, and $\theta_{ij} \equiv \frac{a_{ij}w_i}{p_j}$, the results can be written as

$$\hat{p_1} = \theta_{11}\hat{w}_1 + \theta_{31}\hat{w}_3 \tag{11}$$

$$\hat{p_2} = \theta_{22}\hat{w}_2 + \theta_{32}\hat{w}_3 \tag{12}$$

where $\hat{x}$ is the percentage change of the variable x; θ_{ij} is financial share of input i in good j such that $0 < \theta_{ij} < 1$, $\theta_{11} + \theta_{31} = 1$, and $\theta_{22} + \theta_{32} = 1$.

The total use of agricultural capital, manufacturing capital, and water (i.e. factor constraints) are denoted as V_1, V_2, and V_3, respectively. The key set of equations for the factor markets are:

$$\text{Agcapital: } V_1 = x_{11} = a_{11}(w_1, w_3)y_1 \text{ implies: } y_1 = \frac{V_1}{a_{11}} \tag{13}$$

$$\text{Mcapital: } V_2 = x_{22} = a_{22}(w_2, w_3)y_2 \text{ implies: } y_2 = \frac{V_2}{a_{22}} \tag{14}$$

$$\text{Water}_{agricuture}: x_{31} = a_{31}(w_1, w_3)y_1 \tag{15}$$

$$\text{Water}_{manufacture}: x_{32} = a_{32}(w_2, w_3)y_2 \tag{16}$$

$$\text{Water}_{total}: V_3 = x_{31} + x_{32} = a_{31}y_1 + a_{32}y_2 \tag{17}$$

Eqs. (13), (14), and (17) are the so-called 'factor market clearing' conditions, and imply that a factor is used in its entirety. The market for water connects the two industries. So rather than working with all three equations, we can restate Eq. (17) using the implication of Eqs. (13) and (14):

$$V_3 = \frac{a_{31}}{a_{11}}V_1 + \frac{a_{32}}{a_{22}}V_2 \tag{18}$$

If we totally differentiate and rearrange the equation we obtain the following results (details are in Appendix B):

$$\frac{dV_3}{V_3} = \frac{a_{31}}{a_{11}}\frac{V_1}{V_3}\frac{dV_1}{V_1} + \frac{a_{32}}{a_{22}}\frac{V_2}{V_3}\frac{dV_2}{V_2} + \frac{V_1}{V_3}\frac{a_{31}}{a_{11}}\frac{d\frac{a_{31}}{a_{11}}}{\frac{a_{31}}{a_{11}}} + \frac{V_2}{V_3}\frac{a_{32}}{a_{22}}\frac{d\frac{a_{32}}{a_{22}}}{\frac{a_{32}}{a_{22}}} \tag{19}$$

That is:

$$\hat{V_3} = \lambda_{31}\hat{V_1} + \lambda_{32}\hat{V_2} + \lambda_{31}\left(\hat{a_{31}} - \hat{a_{11}}\right) + \lambda_{32}\left(\hat{a_{32}} - \hat{a_{22}}\right) \tag{20}$$

where the notation of $\hat{x}$ is the same as in Eqs. (11) and (12), which is the percentage change of the variable x; $\lambda_{31} \equiv \frac{a_{31}}{a_{11}}\frac{V_1}{V_3} = \frac{a_{31}y_1}{V_3}$ is the fraction that industry 1 (agriculture) uses of water (input 3); similarly, $\lambda_{32} \equiv \frac{a_{32}}{a_{22}}\frac{V_2}{V_3} = \frac{a_{32}y_2}{V_3}$ is the fraction that industry 2 (manufacturing) uses of water (input 3), such that $\lambda_{31} + \lambda_{32} = 1,\ 0 \le \lambda_{31} \le 1$, and $0 \le \lambda_{32} \le 1$.

Instead of leaving it in terms of the demand for inputs, it is useful to convert to input return changes. To do this, we make use of the elasticity of substitution between the two inputs used in industry 1:

$$\sigma_1 \equiv -\frac{dln(x_{11}/x_{31})}{dln(F_1/F_3)} = -\frac{dln(a_{11}/a_{31})}{dln(w_1/w_3)} = -\frac{\left(\hat{a_{11}} - \hat{a_{31}}\right)}{\left(\hat{w_1} - \hat{w_3}\right)} \tag{21}$$

where F_1 is the partial derivative of the production function (for sector 1) with respect to input 1 ($F_1 = \frac{\partial f_1(x_{11},x_{31})}{\partial x_{11}}$). F_3 is the partial derivative of the production function (for sector 1) with respect to input 3 ($F_3 = \frac{\partial f_1(x_{11},x_{31})}{\partial x_{31}}$).

Similarly, the elasticity of substitution between the two inputs used in industry 2 is:

$$\sigma_2 \equiv \frac{\left(\hat{a_{32}} - \hat{a_{22}}\right)}{\left(\hat{w_2} - \hat{w_3}\right)} \tag{22}$$

The elasticity of substitution measures the substitutability between the two inputs in each industry, which indicates how easy it is to substitute one for the other. A high value of the elasticity of substitution implies that small changes in relative input prices lead to a large shift in input use. In contrast, a small value of the elasticity of substitution implies that changing the relative input prices does not impact input use much. In the limit, where the elasticity of substitution is equal to zero, there is no response to a change in relative input price; this is the case where there is no substitute for water.

Now, rewrite Eq. (20) by substituting Eqs. (21) and (22):

$$\hat{V_3} = \lambda_{31}\hat{V_1} + \lambda_{32}\hat{V_2} + \lambda_{31}\sigma_1(\hat{w_1} - \hat{w_3}) + \lambda_{32}\sigma_2(\hat{w_2} - \hat{w_3}) \tag{23}$$

In summary, the results are:

$$\hat{p_1} = \theta_{11}\hat{w_1} + \theta_{31}\hat{w_3} \tag{24}$$

$$\hat{p_2} = \theta_{22}\hat{w_2} + \theta_{32}\hat{w_3} \tag{25}$$

$$\hat{V_3} - \lambda_{31}\hat{V_1} - \lambda_{32}\hat{V_2} = \lambda_{31}\sigma_1(\hat{w_1} - \hat{w_3}) + \lambda_{32}\sigma_2(\hat{w_2} - \hat{w_3}) \tag{26}$$

Note that Eqs. (24)–(26) are the same as Eqs. (11), (12) and (23) respectively, and are the key equations in the model.

An equivalent way of stating the system is:

$$\begin{pmatrix} \theta_{11} & 0 & \theta_{31} \\ 0 & \theta_{22} & \theta_{32} \\ \lambda_{31}\sigma_1 & \lambda_{32}\sigma_2 & -\lambda_{31}\sigma_1 - \lambda_{32}\sigma_2 \end{pmatrix}\begin{pmatrix} \hat{w_1} \\ \hat{w_2} \\ \hat{w_3} \end{pmatrix} = \begin{pmatrix} \hat{p_1} \\ \hat{p_2} \\ \hat{V_3} - \lambda_{31}\hat{V_1} - \lambda_{32}\hat{V_2} \end{pmatrix} \tag{27}$$

The conditional demand for input by sector $(\hat{a_{11}}, \hat{a_{31}}, \hat{a_{22}}, \hat{a_{32}})$ is variable:

$$\hat{a_{11}} = -\theta_{31}\sigma_1(\hat{w_1} - \hat{w_3}) \tag{28}$$

$$\hat{a_{31}} = \theta_{11}\sigma_1(\hat{w_1} - \hat{w_3}) \tag{29}$$

$$\hat{a_{22}} = -\theta_{32}\sigma_2(\hat{w_2} - \hat{w_3}) \tag{30}$$

$$\hat{a_{32}} = \theta_{22}\sigma_2(\hat{w_2} - \hat{w_3}) \tag{31}$$

Notations are the same as in Eqs. (21) and (22).

Using the differentiated market clearing conditions for inputs 1 and 2, i.e. by differentiating Eqs. (13) and (14), we find that the change in output of the two goods is:

$$\hat{y_1} = \hat{V_1} - \hat{a_{11}} \tag{32}$$

$$\hat{y_2} = \hat{V_2} - \hat{a_{22}} \tag{33}$$

Finally, we can obtain the change in water use of the two sectors by differentiating Eqs. (15) and (16):

$$\hat{x_{31}} = \hat{a_{31}} - \hat{y_1} \tag{34}$$

$$\hat{x_{32}} = \hat{a_{32}} - \hat{y_2} \tag{35}$$

2.2. Decision-making under uncertainty

The stochastic nature of water supplies is another aspect that makes water unique as a factor of production. Under situations of risk and uncertainty in water supplies, decision-makers may act differently than they do when they are solely concerned with maximizing profit. Inter-annual variability in water supplies can be represented by shifts in the supply curve, resulting in price fluctuations. These price fluctuations are due to variability in the water supply, and are what induce behavior on the part of water users that differs from pure profit maximizers in Section 2.2. Now, producers maximize their expected utility under variable water supplies.

Here, we only consider industry 1 since results for industry 2 are analogous. The development parallels that of Howitt and Taylor [13], but, by contrast, is set in a three input, two industry, open economy. The decision rule can be written:

$$\Pi = p_1 f_1(x_{11}, x_{31}) - w_1 x_{11} - w_3 x_{31} \tag{36}$$

where w_3 is the random, imputed value of water, assumed to be normally distributed, $N(\bar{w_3}, \sigma^2)$, and where $\bar{w_3}$ is the expected water price and σ^2 is the variance of the water price.

Further, we assume that firms in industries 1 and 2 have a utility function in profits, maximize expected utility, and are risk averse. Accordingly:

$$\max E[u(\Pi)],\ u'(\Pi) > 0,\ u''(\Pi) < 0 \tag{37}$$

Substituting Eq. (36) into Eq. (37), the decision problem for a representative producer is to maximize expected utility by choosing the input quantity (x_{11} and x_{31}) given the input and output price:

$$\max_{x_{11},x_{31}} E[u(p_1 f_1(x_{11}, x_{31}) - w_1 x_{11} - w_3 x_{31})] \tag{38}$$

The two first order conditions of Eq. (38) that correspond to the two input choices are given by setting the two first order derivatives of Eq. (38) with respect to x_{11} and x_{31}, to be zero:

$$E[u'(\Pi)(p_1\partial f_1/\partial x_{11} - w_1)] = 0 \quad (39)$$

$$E[u'(\Pi)(p_1\partial f_1/\partial x_{31} - w_3)] = 0 \quad (40)$$

The second order conditions of Eq. (38), likewise, are obtained by setting the two second order derivatives to be less than zero:

$$E[u''(\Pi)(p_1\partial f_1/\partial x_{11} - w_1)^2 + u'(\Pi)p_1\partial^2 f_1/\partial x_{11}^2] < 0 \quad (41)$$

$$E[u''(\Pi)(p_1\partial f_1/\partial x_{31} - w_3)^2 + u'(\Pi)p_1\partial^2 f_1/\partial x_{31}^2] < 0 \quad (42)$$

This model framework enables us to derive the level of economic output and water use when farmers face stochastic water supply, which we present in Section 3.4. An intuitive explanation of these results will be given there.

3. Model results

3.1. Scenario 1: agricultural trade liberalization

Scenario 1 evaluates the impacts of agricultural trade liberalization. Suppose the home country enters into a trade agreement in which a foreign country agrees to lower its barriers on agricultural imports from the home country. This makes the agricultural good more affordable to foreign buyers, and they increase their demand for it. This new demand drives up the local price of the agricultural good ($\hat{p}_1 > 0$). We assume a water authority exists that chooses the price of agricultural water supplies (w_3), which is not influenced by trade liberalization negotiations. For example, it is unlikely that an irrigation district in California would change the price of water charged to users due to trade negotiations; however producers may have a higher demand for water should the price of their commodity increase. To represent the fact that the price of water is set by an external agency and is held constant, the price of agricultural water is an exogenous variable and is held constant ($\hat{w}_3 = 0$) in this scenario.

The total use of agricultural capital and manufacturing capital are also exogenous and held constant in this scenario ($\hat{V}_1 = 0$, $\hat{V}_2 = 0$), since they will not necessarily be influenced by free trade. This approach could be varied in future work, but in this case enables us to evaluate the pure effect of trade liberalization on water resources controlling for investment. It is possible that the region will invest more capital in response to the new demand following trade liberalization, but this would be a policy response under our model framework, in which $\hat{V}_1$ would need to be adjusted to introduce this.

In particular, the exogenous variables here are:

$$\hat{p}_1 > 0, \hat{p}_2 = 0, \hat{V}_1 = \hat{V}_2 = \hat{w}_3 = 0$$

while the endogenous variables are:

$$\hat{w}_1, \hat{w}_2, \hat{V}_3$$

Under the imposed values, we can apply the control equations in our model (i.e. Eqs. (24)–(35) to obtain values for all other variables in terms of percentage changes. These results are summarized in Table 1.

Total water use under agricultural trade liberalization is given by:

$$\hat{V}_3 = \frac{\lambda_{31}\sigma_1}{\theta_{11}}\hat{p}_1 > 0 \quad (43)$$

The unit water use in agriculture is given by:

$$\hat{a_{31}} = \sigma_1\hat{p}_1 > 0 \quad (44)$$

So, according to Eq. (43) the percentage increase in water use ($\hat{V}_3$) is positively correlated with the fraction of water use in agriculture ($0 < \lambda_{31} < 1$), the elasticity of substitution between the two inputs ($\sigma_1 > 0$), and the percentage increase in the price of agriculture ($\hat{p}_1 > 0$). The percentage increase in water use is negatively correlated with the financial share of agricultural capital ($0 < \theta_{11} < 1$). Under trade liberalization, $\hat{V}_3 > 0$ (refer to Table 1), indicating that domestic water use increases. Note that $\hat{x_{31}} > 0$ and $\hat{x_{32}} = 0$ (refer to Table 1), indicating that the entire increase comes from agriculture. Parameters in the production function determine the extent

Table 1
Percentage change in the model parameters for Scenarios 1, 2, and 3.

	Variables		Scenario 1	Scenario 2	Scenario 3
Resource use	Agcapital	$\hat{V}_1$	0	0	> 0
	Mcapital	$\hat{V}_2$	0	0	0
	Water_{total}	$\hat{V}_3$	$\frac{\lambda_{31}\sigma_1}{\theta_{11}}\hat{p}_1$	0	0
	$\text{Water}_{agriculture}$	$\hat{x_{31}}$	$\frac{\sigma_1}{\theta_{11}}\hat{p}_1$	$\{\frac{\sigma_1}{\theta_{11}}(\hat{p}_1 - \hat{w}_3^*)$	> 0
	$\text{Water}_{manufacture}$	$\hat{x_{32}}$	0	$-\frac{\sigma_2}{\theta_{22}}\hat{w}_3^* < 0$	< 0
Output price	Agriculture	$\hat{p}_1$	$\hat{p}_1 > 0$	$\hat{p}_1 > 0$	0
	Manufacturing	$\hat{p}_2$	0	0	0
Input price	Agcapital	$\hat{w}_1$	$\frac{\hat{p}_1}{\theta_{11}} > \hat{p}_1$	$\frac{\hat{p}_1}{\theta_{11}} - \frac{\theta_{31}}{\theta_{11}}\hat{w}_3^* > \hat{p}_1$	< 0
	Mcapital	$\hat{w}_2$	0	$-\frac{\theta_{32}}{\theta_{22}}\hat{w}_3^* < 0$	< 0
	Water	$\hat{w}_3$	0	$\frac{1}{1+\frac{\lambda_{32}\sigma_2/\theta_{22}}{\lambda_{31}\sigma_1/\ 11}}\hat{p}_1 < \hat{p}_1$	> 0
Conditional demand	Agcapital	$\hat{a_{11}}$	$-\frac{\theta_{31}\sigma_1}{\theta_{11}}\hat{p}_1 < 0$	$-\frac{\theta_{31}\sigma_1}{\theta_{11}}(\hat{p}_1 - \hat{w}_3^*) < 0$	> 0
	Water in agriculture	$\hat{a_{31}}$	$\sigma_1\hat{p}_1 > 0$	$\sigma_1(\hat{p}_1 - \hat{w}_3^*) > 0$	< 0
	Mcapital	$\hat{a_{22}}$	0	$\frac{\theta_{32}\sigma_2}{\theta_{22}}\hat{w}_3^* > 0$	> 0
	Water in manufacturing	$\hat{a_{32}}$	0	$-\sigma_2\hat{w}_3 < 0$	< 0
Quantity of supply	Agriculture	$\hat{y}_1$	$\frac{\theta_{31}\sigma}{\theta}$	$\frac{\theta_{31}\sigma_1}{\theta_{11}}(\hat{p}_1 - \hat{w}_3^*) > 0$	> 0
	Manufacturing	$\hat{y}_2$		$-\frac{\theta_{32}\sigma_2}{\theta_{22}}\hat{w}_3^* < 0$	0

Notes: Those in gray are the exogenous variables. $\hat{x} \equiv \frac{dx}{x}$ is the percentage change of the variable x. $\theta_{ij} \equiv \frac{a_{ij}w_i}{p_j}$ is financial share of input i in goods j. $\lambda_{31} \equiv \frac{a_{31}}{a_{11}}\frac{V_1}{V_3} = \frac{a_{31}y_1}{V_3}$ is fraction that industry 1 (agriculture) uses of water (input 3); similarly, $\lambda_{32} \equiv \frac{a_{32}}{a_{22}}\frac{V_2}{V_3} = \frac{a_{32}y_2}{V_3}$ is fraction that industry 2 (manufacturing) uses of water (input 3). σ_j is elasticity of substitution between the two inputs in industry j.$\hat{w}_3^* = \frac{1}{1+\frac{\lambda_{32}\sigma_2}{\theta_{22}}/\frac{\lambda_{31}\sigma_1}{\theta_{11}}}\hat{p}_1 < \hat{p}_1$ is the target tax on water use.

to which agricultural water use increases in relation to the price of agriculture.

Data for Morocco – a water-scarce country – can help us to narrow down the value of key parameters that determine the extent to which agricultural water use increases under trade liberalization. In Morocco, approximately 87% of total freshwater withdrawals goes towards agricultural production [29], which provides a rough estimate of the fraction of water use in agriculture ($\lambda_{31} \approx$ 87%). The cost share of water use in Morocco ranges from 0.2% to 26% of total input costs [27]. That is, the financial share of agcapital in agriculture varies from 74% to 99.8% (74% $< \theta_{11} <$ 99.8%). So, the total water use increase depends largely on the elasticity of substitution (σ_1).

Conversely, the use of agcapital will increase by proportionately less than water, illustrated by the results $\hat{a_{11}} < 0$ while $\hat{a_{31}} > 0$ (refer to Table 1). In fact, an increase in water use per unit of agricultural production ($\hat{a_{31}} > 0$) suggests a decrease in agricultural water use efficiency. This means that more water resources are used rather than capital-intensive water-saving technologies, such as sophisticated irrigation equipment. This is because the price of capital in agriculture (w_1) increases, without a subsequent increase in the price of water.

As discussed above, total water use ($\hat{V_3}$) and unit agricultural water use ($\hat{a_{31}}$) are strongly influenced by the substitution elasticity (σ_1). σ_1 reflects the ability of capital to be substituted for water, when water is either unavailable or relatively expensive. Potential values of σ_1 range from zero to infinity, which results in a broad range of $\hat{V_3}$. σ_1 equals infinity when water and capital are perfect substitutes in production: if water is in short supply, we can fully compensate for it by using more of another input. If σ_1 equals zero, then there is no substitute for water. In this case, free trade has no effect on total water use ($\hat{V_3} = 0$). Both of these boundary values for σ_1 are unrealistic: it is more likely that some substitution with water is possible. The Leontief assumption might underestimate the effect of trade liberalization on total water use. This is because the Leontief assumption assumes a small elasticity of substitution, such that trade liberalization will result in only a modest increase in the total water use. This is consistent with findings in the literature (e.g. Calzadilla et al. [5]). A more realistic estimate may be the well-known Cobb–Douglas assumption, in which $\sigma_1 = 1$. When $\sigma_1 = 1$ the total water use will increase in proportion to the price of agriculture.

Trade liberalization that raises the agricultural price – without subsequent price adjustments in water – has no effect on manufacturing; all the parameters in industry 2 remain unchanged. Agricultural output increases in order to meet the new demand of the foreign country under trade liberalization, which improves the economic performance of producers in the home country. However, these economic gains come at the expense of water resources; water resource use increases and agricultural water-use efficiency declines. It should be noted that these results depend on the model assumptions that we have made; different assumptions could be made that may lead trade liberalization to impact water use in a different way. We have attempted to provide a framework for such future investigations.

3.2. Scenario 2: target tax on water

Total water use increased under trade liberalization in Scenario 1. If viewed as undesirable, one potential policy response to reduce this increase in water use could be to increase its price. Here, we explore if a water tax exists that reduces water use without offsetting the economic gains from trade liberalization. We derive the level of a tax on water that would counteract the increased water use under trade liberalization. The tax level targets a certain level of water use, so we refer to this tax level as a 'target tax'.

In this scenario, there is agricultural trade liberalization, with increased foreign demand for the water-intensive agricultural good ($\hat{p_1} > 0$). The water authority adjusts w_3 such that water use will remain at pre-liberalization levels, that is, V_3 is held constant; $\hat{V_3} = 0$. The water authority can act such that, in effect, the water use charge (w_3) is an endogenous variable. Allowing w_3 to be endogenous is a way of imputing what its tax might be to return water use to pre-liberalization levels.

In this scenario, the exogenous variables are:

$$\hat{p_1} > 0, \hat{p_2} = 0, \hat{V_1} = \hat{V_2} = \hat{V_3} = 0$$

while the endogenous variables are:

$$\hat{w_1}, \hat{w_2}, \hat{w_3}$$

If we apply the control equations in our model (e.g. Eqs. (24)–(35), we obtain results for all other variables. These are summarized in Table 1. Results for Scenario 2 in Table 1 show the percent change in parameters compared to a base case with no trade liberalization and no water tax. Differences between the columns for Scenario 1 and Scenario 2 in Table 1 indicate the direct impact of the water tax under trade liberalization.

From Table 1, it is clear that agricultural production increases, but not by as much as it does under Scenario 1. This is due to the increased price of water under the target tax in Scenario 2. In Scenario 2, the total water use does not change. However, note that water use increases in agriculture ($\hat{x_{31}} > 0$) are offset by declining water use in manufacturing ($\hat{x_{32}} < 0$) to keep total water use constant. In manufacturing, capital tends to be substituted for water, since water is relatively more expensive now ($\hat{w_3} > 0,\ \hat{w_2} < 0$).

These results illustrate that the target tax is proportionately smaller than increases in agricultural prices under trade liberalization. This framework provides a way to estimate the target tax, which is given by:

$$\hat{w}_3^* = \frac{1}{1 + \frac{\lambda_{32}\sigma_2}{\theta_{22}} / \frac{\lambda_{31}\sigma_1}{\theta_{11}}} \hat{p_1} < \hat{p_1} \tag{45}$$

Note that the target tax, $\hat{w}_3^*$, depends on the fraction of water use, substitution elasticity of inputs, and financial share of capital between manufacturing and agriculture ($\frac{\lambda_{32}}{\lambda_{31}}, \frac{\sigma_2}{\sigma_1}$, and $\frac{\theta_{22}}{\theta_{11}}$, all of which are positive). Thus, the finding that $\hat{w_3} < \hat{p_1}$ demonstrates that a tax – that is proportionately smaller than the percentage increase in the agricultural price ($\hat{p_1}$) – is large enough to offset the increase of water use resulting from trade liberalization.

Fig. 2 illustrates the percentage change in model parameters for various values of the water tax. Fig. 2 highlights the increasing intersectoral competition for water as water becomes more valuable. Water use in both sectors falls as the price of water increases, yet at different rates, determined by its elasticity of substitution (σ_1 and σ_2) and financial share of capital (θ_{11} and θ_{22}). Total water use returns to pre-liberalization levels ($\hat{V_3} = 0$) at the target tax $\hat{w}_3^*$, by definition. However, these reductions in water use are accompanied by further reductions in agricultural output, until the gains from trade are completely eliminated when $\hat{w_3} = \hat{p_1}$, since $\hat{y_1} = 0$ (refer to Fig. 2d). Additionally, $\hat{y_2} < 0$ for all tax levels (refer to Fig. 2e). In other words, for the tax level $\hat{w_3} = \hat{p_1}$ agricultural output returns to pre-liberalization levels, while manufacturing output is reduced for any level of the tax, indicating reduced economic performance. Thus, tradeoffs exist between economic performance and resource use – which must be carefully considered – in order to determine implications of the target tax.

3.3. Scenario 3: subsidize investment in agriculture

In this scenario, we explore another potential policy option to reduce water use. In this case there is additional investment in the

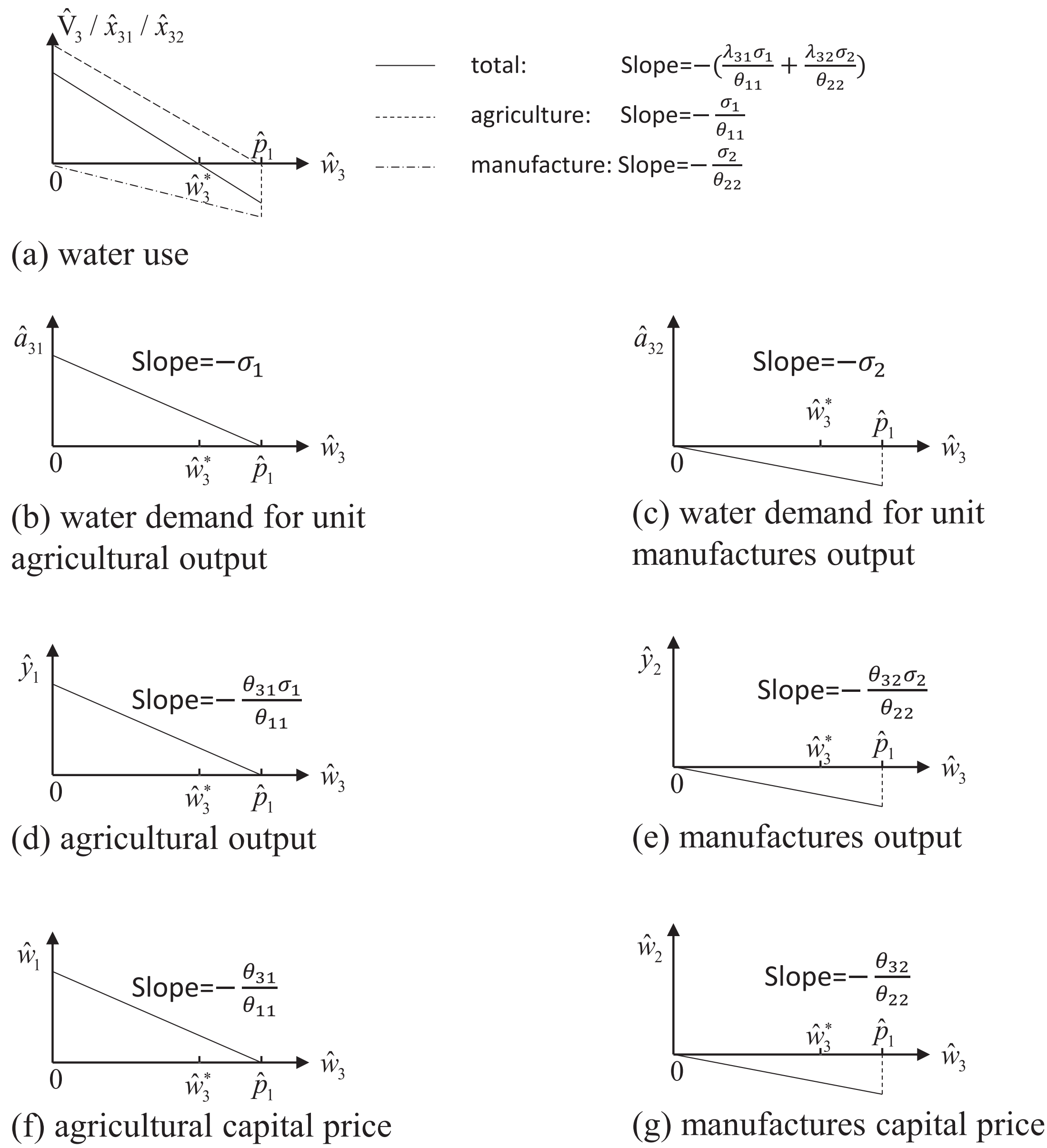

Fig. 2. Percentage change in the model parameters with the water tax ($\hat{w}_3$). When $\hat{w}_3 = 0$, then no water tax is present and only trade liberalization holds. The target tax on water is shown by $\hat{w}_3^*$. The boundary value for the tax ($\hat{p}_1$) indicates the value of the tax for which agricultural price increases due to liberalization are eliminated. The model parameters shown are the percentage change in (a) water use, (b) water demand for unit agricultural output, (c) water demand for unit manufacturing output, (d) agriculture output, (e) manufacturing output, (f) agricultural capital price, and (g) manufacturing capital price. Water use returns to pre-liberalization levels at the target tax by definition ($\hat{V}_3 = 0$). At tax levels greater than $\hat{w}_3^*$, water use continues to fall, as does agricultural output, until the gains from trade are completely eliminated when $\hat{w}_3 = \hat{p}_1$, since $\hat{y}_1 = 0$.

agriculture sector, which might arise from government interventions that are external to the scope of the model. Not all investment in agricultural capital will reduce water use; we restrict our attention to those that do, such as water-saving technologies.

Here, the use of capital in agriculture is increased, such that $\hat{V}_1 > 0$. In this scenario, there is no trade liberalization, so $\hat{p}_1 = 0$, because there is no additional foreign demand for agricultural goods. So, the exogenous variables in this scenario are:

$$\hat{p}_1 = 0, \hat{p}_2 = 0, \hat{V}_1 > 0, \hat{V}_2 = \hat{V}_3 = 0$$

while the endogenous variables are:

$$\hat{w}_1, \hat{w}_2, \hat{w}_3$$

Under the imposed values, we apply the control equations in our model (e.g. Eqs. (24)–(35) and obtain results for all other variables. These are summarized in Table 1. Results for Scenario 3 in Table 1 show the percent change in parameters compared to a base case with no trade liberalization and no subsidy to agcapital.

The finding that $\hat{w}_3 > 0$ implies that water has become relatively expensive in comparison to agricultural capital. Returns to specific inputs (e.g. cost of agricultural capital and manufacturing capital) fall to offset the increase in water price, shown by $\hat{w}_1 < 0$ and $\hat{w}_2 < 0$ (refer to Table 1). Notably, agricultural production increases under this scenario (Note that $\hat{y}_1 > 0$ under Scenario 3 in Table 1). Since agricultural production increases while water use remains fixed, the unit water use decreases (i.e. water-use efficiency increases). Since water-saving technologies become cheaper in this scenario, producers substitute away from water, which might be the goal of the public intervention.

An implication of this scenario is that mechanisms exist by which producers use less water without reducing agricultural production. It is thus an indirect means of influencing water use decision-making. Subsidizing agricultural capital – combined with other relevant policies – provides a policy mechanism to reduce water use without targeting water resources explicitly. One downside is that would shift government resources away from other activities, and a complete analysis would need to examine how the capital investment is paid for. A broader, more detailed general equilibrium model with an explicit role for government and public finance would be needed to fully explore this issue.

3.4. Scenario 4: stochastic water supplies

In this scenario, water supplies are stochastic, which is an important feature of water resources. Variability in water supplies can result in variability in the price paid for water (w_3) as discussed in Section 2.2. For example, a farmer may have riparian rights for a river whose flow varies from year to year. The rental value of water also will have a distribution in this case. Previous studies have examined the impact of variable water input prices on the decision to adopt precision irrigation technology [24], but have not yet considered trade implications.

Here, we apply the model described in Section 2.2 and obtain the following results (see details in Appendix C):

$$p_1 \partial f_1 / \partial x_{11} = w_1 \quad (46)$$

$$p_1 \partial f_1 / \partial x_{31} > \bar{w_3} \quad (47)$$

Eq. (46) is a standard result in a competitive market setting. This result highlights the fact that the value marginal product of input 1 is equivalent to the exogenously determined price of input 1, which is the same as the result in equilibrium without a stochastic water price.

Eq. (47) provides a finding similar to Howitt and Taylor [13], but is an extension to a trade setting, with more than one sector and with multiple factors of production. The left hand side (LHS) of Eq. (47) presents the agricultural output multiplied by the extra agricultural output that can be derived from one more unit of water. The right hand side (RHS) of Eq. (47) presents what must be paid for that water. In the absence of water risk, the LHS and RHS would be equal (as in the previous equation). This would mean that the producer would keep procuring more water up to the point that the extra revenue from it equals the cost of getting the water. However, what we show under stochastic supplies and risk aversion, is that the producer does not obey this rule. Instead, they have to be guaranteed a return in excess of what they pay for the water (on average). In essence the producer is giving up potential revenue, on average.

Thus, Eq. (47) suggests expected utility is maximized when the value marginal product exceeds the expected factor cost. This is a very important result. It means that producers will cut back on water use simply because of uncertainty. The additional revenue derived from obtaining more water must be strictly greater than the price of obtaining more water, in order to induce producers to make this expansion.

The above result is based on the assumption that producers are risk averse and does not hold if producers are not risk averse. However, we do not make any assumptions about their level of risk aversion. The amount that producers cut back on water use is associated with their level of risk aversion and may also be distorted by existing policies, such as insurance contracts. Producer levels of risk aversion remains an empirical question that we necessarily leave to future work. What makes this scenario different from the other scenarios is that water availability is stochastic and producers are not indifferent to the associated risks. It implies there is a behavioral response to stochastic water supplies that leads to a reduction in water use. In this way, this approach is more general than the profit-maximizing assumption used to derive earlier results. Note that when water supplies are not stochastic, the expected utility maximization behavioral approach of Section 2.2 yields equivalent results to that of profit maximization. In fact, we could have used the (more complicated) expected utility maximization approach through the paper, with no changes in results.

4. Conclusions

In this paper, we presented a model that captures the mechanistic relationships between water as a domestic factor of production, as well as the consumption and trade of water-intensive commodities. We focus on four tradeoffs that are important in water-use decision-making. The first tradeoff concerns water allocation among different sectors. The second tradeoff highlights substitution possibilities in production, which means that, under certain circumstances, there may be a substitute for water (or at least a way to conserve water), such as water-saving technologies that require capital investments. The third tradeoff highlights that production and consumption can be substituted across regions, in which trade lends flexibility and efficiency to the system. The fourth tradeoff considers how farmers change their behavior to account for production risk.

We show that inclusion of stochastic water supplies into the model induces producers to reduce their demand for water, which is an unclear relationship ex-ante. Other results show the conditions under which agricultural trade liberalization influences water use, depending on the elasticity of substitution of water. An important result is that a target tax – which offsets the increase in water consumption due to trade liberalization – can be proportionately smaller than the increase in agricultural output price. Subsidizing capital in the agricultural sector may lead to the adoption of advanced technologies, such as drip irrigation and water saving seed and fertilizer technologies, which increases water-use efficiency and reduces water consumption in agriculture. This highlights the fact that policies that are not directly focused on water resources may have a significant impact on water use. This may be helpful in situations in which direct water policies are politically unpalatable.

The model that we presented in this paper contributes to the further integration of the hydro-economics, socio-hydrology, and virtual water trade literatures. The motivation of our model was to develop generalizable insights into the interactions between people and water resources. In this way, our approach parallels that of socio-hydrology, while broadening socio-hydrology by explicitly incorporating economics. Our theoretical model complements existing hydro-economics models, which are typically parameterized to capture local dynamics with the goal of improving water resources management. Additionally, our model provides a theoretical foundation for virtual water trade research.

The strength of our analysis has been to capture the tradeoffs that exist for water use decision-making between sectors, factors, trading partners, and with uncertainty. Significantly, our model enables these linkages and mechanisms to be displayed in a simple and coherent framework, in which all assumptions are clearly stated. That said, our analysis has left unexamined a number of issues that may be important in certain circumstances. Future extensions to the model may consider the implications of infrastructure and distinct sources of water, such as rainfall, surface irrigation supplies, and groundwater supplies. The integration of theory with empirical analyses represents a particularly important direction for future research.

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Appendix A

Eqs. (7) and (8) are as the following:

$$p_1 = w_1 a_{11}(w_1, w_3) + w_3 a_{31}(w_1, w_3) \tag{A.1}$$

$$p_2 = w_2 a_{22}(w_2, w_3) + w_3 a_{32}(w_2, w_3) \tag{A.2}$$

Now, totally differentiate the competitive profit condition (Eqs. (A.1) and (A.2)):

$$dp_1 = a_{11}dw_1 + a_{31}dw_3 + w_1 da_{11} + w_3 da_{31} = a_{11}dw_1 + a_{31}dw_3 \tag{A.3}$$

$$dp_2 = a_{22}dw_2 + a_{32}dw_3 + w_2 da_{22} + w_3 da_{32} = a_{22}dw_2 + a_{32}dw_3 \tag{A.4}$$

Since $w_1 da_{11} + w_3 da_{31} = 0$ and $w_2 da_{22} + w_3 da_{32} = 0$. This result arises from the so-called 'envelope theorem' of economics, which concerns the differentiability properties of the objective function – in this case a firm's optimization problem. Divide both sides by p_1 in Eq. (A.3) and p_2 in Eq. (A.4), and multiply/divide RHS terms by certain terms:

$$\frac{dp_1}{p_1} = \frac{a_{11}w_1}{p_1}\frac{dw_1}{w_1} + \frac{a_{31}w_3}{p_1}\frac{dw_3}{w_3} \tag{A.5}$$

$$\frac{dp_2}{p_2} = \frac{a_{22}w_2}{p_2}\frac{dw_2}{w_2} + \frac{a_{32}w_3}{p_2}\frac{dw_3}{w_3} \tag{A.6}$$

Thus we obtain Eqs. (9) and (10).

Appendix B

Eq. (18) is as the following:

$$V_3 = \frac{a_{31}}{a_{11}}V_1 + \frac{a_{32}}{a_{22}}V_2 \tag{B.1}$$

Now totally differentiate Eq. (B.1):

$$dV_3 = \frac{a_{31}}{a_{11}}dV_1 + \frac{a_{32}}{a_{22}}dV_2 + V_1 d\frac{a_{31}}{a_{11}} + V_2 d\frac{a_{32}}{a_{22}} \tag{B.2}$$

Divide both sides by V_3 in Eq. (B.1):

$$\frac{dV_3}{V_3} = \frac{a_{31}}{a_{11}}\frac{V_1}{V_3}\frac{dV_1}{V_1} + \frac{a_{32}}{a_{22}}\frac{V_2}{V_3}\frac{dV_2}{V_2} + \frac{V_1}{V_3}\frac{a_{31}}{a_{11}}\frac{d\frac{a_{31}}{a_{11}}}{\frac{a_{31}}{a_{11}}} + \frac{V_2}{V_3}\frac{a_{32}}{a_{22}}\frac{d\frac{a_{32}}{a_{22}}}{\frac{a_{32}}{a_{22}}} \tag{B.3}$$

Thus we obtain Eq. (19).

Appendix C

Eqs. (39) and (40) are as the following:

$$E[u'(\Pi)(p_1\partial f/\partial x_{11} - w_1)] = 0 \tag{C.1}$$

$$E[u'(\Pi)(p_1\partial f/\partial x_{31} - w_3)] = 0 \tag{C.2}$$

The expected value operator passes over all terms in the uppermost first order condition (Eq. (C.1)) because no term is random. This condition can be simplified to be:

$$p_1\partial f/\partial x_{11} = w_1 \tag{C.3}$$

Here, we get Eq. (46).

The first order condition with respect to water is more complicated because the price has a distribution. We first rearrange Eq. (C.2):

$$E[u'(\Pi)p_1\partial f/\partial x_{31}] = E[u'(\Pi)w_3] \tag{C.4}$$

Now subtract $E[u'(\Pi)\bar{w}_3]$ from both sides yields in Eq. (C.4) and simplify:

$$E[u'(\Pi)(p_1\partial f/\partial x_{31} - \bar{w}_3)] = E[u'(\Pi)(w_3 - \bar{w}_3)] \tag{C.5}$$

The value of profit is as in Eq. (36):

$$\Pi = p_1 f_1(x_{11}, x_{31}) - w_1 x_{11} - w_3 x_{31} \tag{C.6}$$

If we take the expected value of profit in Eq. (C.6), we get:

$$E(\Pi) = p_1 f(x_{11}, x_{31}) - w_1 x_{11} - \bar{w}_3 x_{31} \tag{C.7}$$

Substituting Eq. (C.6) into Eq. (C.7), we get:

$$\Pi = E(\Pi) + (\bar{w}_3 - w_3)x_{31} \tag{C.8}$$

If $\bar{w}_3 > w_3$ then $\Pi > E(\Pi)$ and from the properties of the utility function $u'(\Pi) < u'(E(\Pi))$ (risk aversion) and therefore:

$$u'(\Pi)(\bar{w}_3 - w_3) < u'(E(\Pi))(\bar{w}_3 - w_3) \tag{C.9}$$

If $\bar{w}_3 < w_3$ then $\Pi < E(\Pi)$ and then: $u'(\Pi) > u'(E(\Pi))$. However we get the same relationship as in Eq. (C.9):

$$u'(\Pi)(\bar{w}_3 - w_3) < u'(E(\Pi))(\bar{w}_3 - w_3) \tag{C.10}$$

since the sign of $\bar{w}_3 - w_3$ is changed.

Therefore, Eq. (C.10), which is the same as Eq. (C.9), holds for all $\bar{w}_3$ and w_3. Taking expectations of both sides of Eq. (C.10), we get:

$$E[u'(\Pi)(\bar{w}_3 - w_3)] < u'(E(\Pi))E(\bar{w}_3 - w_3) \tag{C.11}$$

Since $E(\bar{w}_3 - w_3) = 0$, substituting this into Eq. (C.11), we then have that:

$$E[u'(\Pi)(\bar{w}_3 - w_3)] < 0 \tag{C.12}$$

Substituting Eq. (C.5) into Eq. (C.12), we obtain

$$E[u'(\Pi)(p_1\partial f/\partial x_{31} - \bar{w}_3)] > 0 \tag{C.13}$$

which implies Eq. (47) at the optimum:

$$p_1\partial f/\partial x_{31} > \bar{w}_3 \tag{C.14}$$

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